

Continuum view of Power Grids using angle distance

C.H.WELLS
OSIsoft, LLC
United States of America
cwells@osisoft.com

KEYWORDS

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1 INTRODUCTION

A new method of computing angle distances for use in Wide Area Measurement (WAM) system performance is described. From this, unambiguous angle differences are computed. The concept is supported using a continuum view of the electric power grid. The wide area system is considered to be described by a set of partial differential equations rather than the traditional view of differential-algebraic equations. This approach explains observed behavior both in the Eastern Interconnection and the WECC.

The measurements are collected using a cyber-secure synchrophasor platform (CSSP) installed at the University of California, San Diego (UCSD) sensing low voltage single phase circuits on the local microgrid. The CSSP consists of redundant phasor measurement units (PMUs) with dual substation-hardened computers. The purpose of the CSSP is to provide a secure base system for use in real time computations for power system as well as providing local visibility for substation personnel and supporting control system calculations providing UCSD facility operators with information to prepare for a potential island maneuver. This configuration allows the system to be patched as required by the North America Electric Reliability Council (NERC) while still continuing to collect necessary data for real time computation (NERC CIP) requirements.

The CSSP includes a short-term archive of PMU synchrophasor data and real-time computations of Fast Fourier Transforms (FFTs) and damping coefficients at each mode. Angle separation and FFTs of the data are used to identify a potential island maneuver that might be executed.

2 BODY

2.1 Problem definition

Angle measurements from PMUs meeting IEEE C37.118.1 standards are reported over the range $\pm \pi$ radians or ± 180 degrees and “wrap” when the reported angle exceeds this range. The frequency offset from nominal system frequency (60 or 50 Hz) is equal to the rate of change of the angle, so if the system frequency is exactly 60 Hz, the reported phasor angle will remain constant. Voltage and current angles are measured relative to a “virtual bus” operating at exactly 60 Hz with the peak of its cosine wave coincident with the “top of second.” The angle trend line has a slope directly proportional to the offset from the nominal frequency. A typical example is shown below in Fig. 1.

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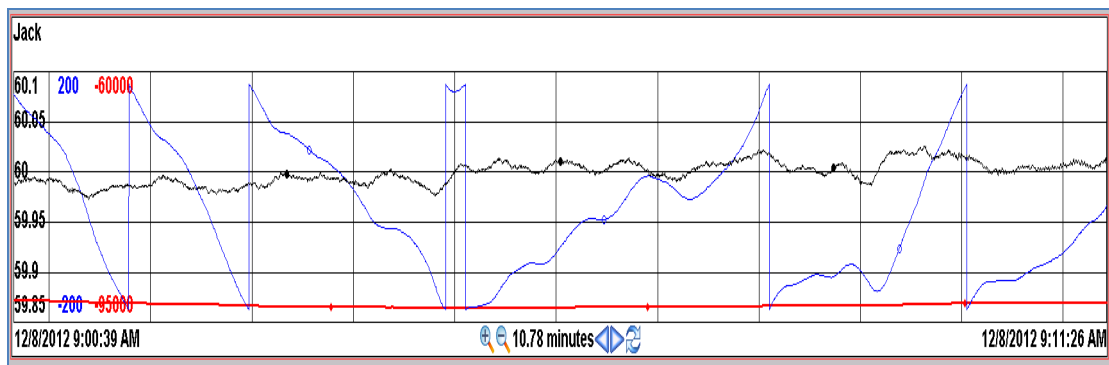


Fig. 1: Wrapped angle and frequency over 11 minute time range.

The blue line is the wrapping voltage angle of a single phase measured from a 120 volt wall outlet in Jacobs Hall in UCSD, black is frequency in Hz, and red is accumulated angle distance in degrees which will be described below (this will be referred to as angle distance). There are three scales in this Fig.: frequency 59.85 to 60.1 Hz, wrapped angle from -200 to +200 degrees and angle distance from -95,000 to -60,000 degrees. This is the total accumulation of degrees of deviation from the IEEE C37.118 reference angle since midnight: this could also be reported as cycles. Note that the angle wrapped six times (three negative wraps and three positive within this interval and the slope changed from negative to positive indicating that the frequency was initially less than 60 Hz and then became greater than 60 Hz. When the frequency is exactly 60 Hz, the slope of the angle (rate of change of angle with respect to time) is zero indicating that the phasor angle is stationary.

The red line in Fig. 1 shows that the angle distance from the IEEE C37.118 reference is nearly 95,000 degrees below the reference. This translates to about 4.39 seconds slow or about 264 cycles behind the standard. The angle distance is reset at midnight each day. At reset, the angle distance from the reference is equal to its current wrapped value.

A clarifying example is shown in Fig. 2. The frequency scale is the same as in Fig. 1 but horizontal axis covers a one day interval the cumulative angle distance ranges from -200,000 degrees to +100,000 degrees.

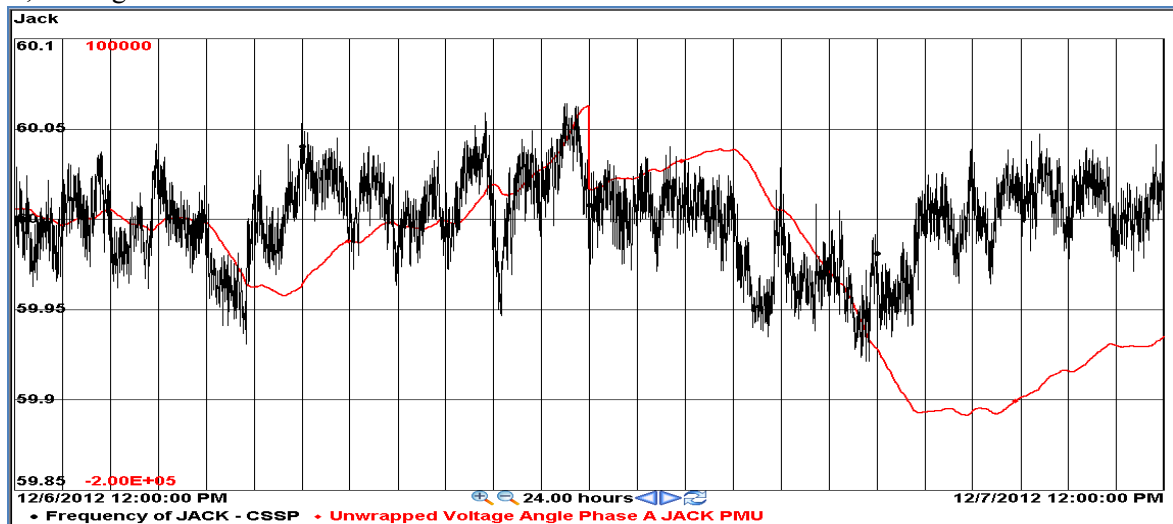


Fig. 2: Accumulated angle distance and frequency

This shows angle distance and frequency over a 24 hour period beginning at noon and ending at noon the next day. The angle distance is reset to its current wrapped value at exactly midnight (center of the graph) (this is the raw angle measurement directly from the PMU). This also shows that the accumulated angle does not return to its starting point after 24 hours as one would expect. For example, the angle at noon on December 6 is -12,973 degrees and at noon on December 7 is -100,206

degrees showing a difference of -87,233 degrees which translates to 243 cycles or 4.03 seconds. This means that over this 24 hour period the grid lost 4.03 seconds and was 243 cycles behind the reference bus, (the IEEE C37.118 bus angle reference). The same behavior is observed if the daily interval begins and ends at midnight.

Subtraction of two discontinuous angles results in a discontinuous signal and since power flow is a function of the angle difference between two buses, the power flow direction could be ambiguous (see below). For example in Fig. 1, any angle subtracted from this angle trend will also be discontinuous. The PMU angle measurements must be unwrapped to provide a smooth difference calculation.

Angles can be unwrapped either before or after subtraction. But these operations are not commutative. The argument is made case that subtraction should be done after the angles are unwrapped: that is, unwrap first then subtract. Details are described below, but what follows is a discussion of why unwrapped angles (angular distance from the standard) are important in grid observations and wide area control.

Generally, the grid is considered a synchronous machine with multiple interconnected generators and loads. It is commonly modeled with sets of differential-algebraic equations (DAE) called power flow equations, where the algebraic portion represents the transmission lines and the generators and loads are represented by ordinary differential equations that represent their inertial components. The equation sets are formed by writing voltage and current equations around each bus using Kirchoff's laws. A set of non-linear differential equations constrained by algebraic equations are formed and since we can only write (n-1) equations, one bus in the system must be defined with zero or the reference angle. This is often called the slack bus. The power flow equations are solved by iterative techniques that involve matrix inversion of the normally sparse admittance matrix. The solution of the DAEs depends on the initial condition and infeasible solutions can result based on a poor set of initial conditions.

An alternate approach is to consider the grid as a continuous system that includes both temporal and spatial aspects. This is a relatively new approach to grid analysis but has been covered extensively in the literature. The most commonly referenced work is that by Thorp and Phadke and their students. A recent PhD dissertation by one of Phadke's students [1] outlines a novel approach to simulation of the continuum representation of the grid. This shows explicit formation of electromechanical waves in the grid. This phenomenon was clearly observed in the Turkey Point event analysis video produced by Virginia Tech².

In the continuum approach, one set of partial differential equations (PDE) are used to represent the spatial transmission effects and a second set represents the continuum behavior of the generators and loads. The set of PDEs require boundary conditions. Most simplified forms of the PDEs exhibit traveling wave behavior. Thus a disturbance at any point in the grid shows the formation and propagation of a traveling wave over a relative long distance and time frame. For example in the Turkey Point event, the wave persisted for at least 13 seconds and traveled from South Florida to Canada and reflected back to Florida several times. Clearly the frequency of generators in South Florida were operating at different speeds than those in Canada and their angle differences could have 360 degrees.

An example of wave formation from Liling thesis is shown below in Fig. 3. She used a simplified solution to the PDEs to obtain these results.

¹ Liling.. http://scholar.lib.vt.edu/theses/available/etd-10162003-125452/unrestricted/Liling_Dissertation.pdf

² A snapshot of this video is shown in the next section.

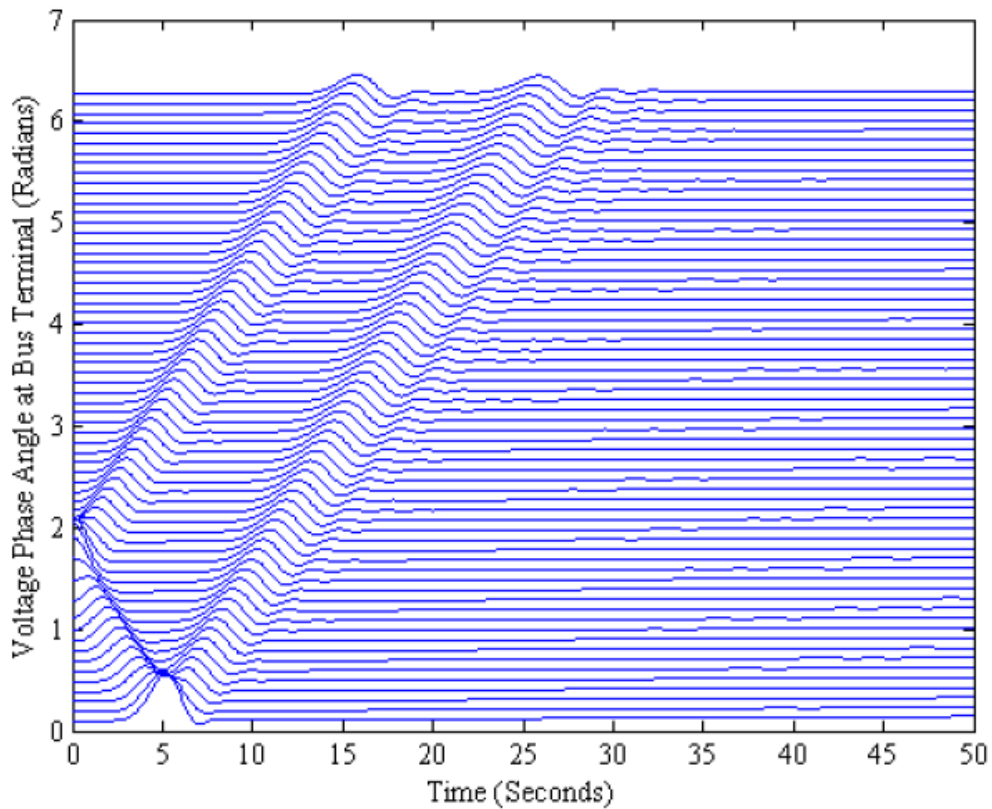


Fig. 3: Simulation of wave propagation in the power grid, by Liling.

This was confirmed by observations of the Turkey Point Event occurring in 2008 as shown below in Fig. 4.

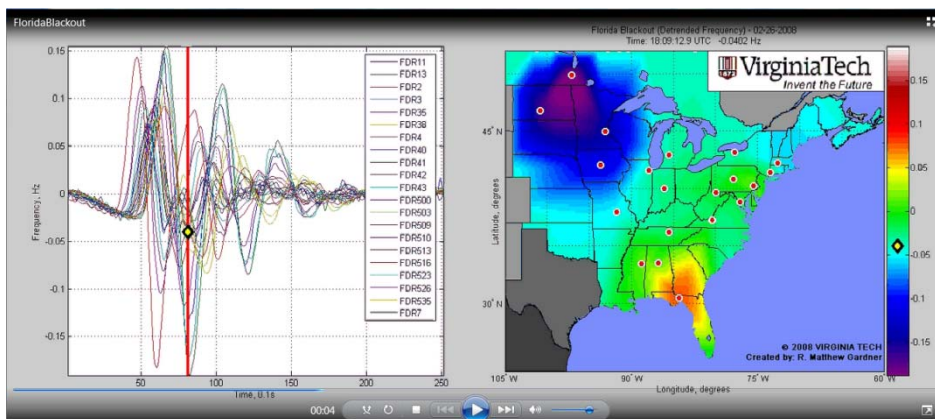


Fig. 4: Turkey Point Oscillation in the Eastern Interconnection, courtesy of Virginia Tech.

Note the time scale of the video. This is consistent with the actual wave propagation observed in the Turkey Point event.

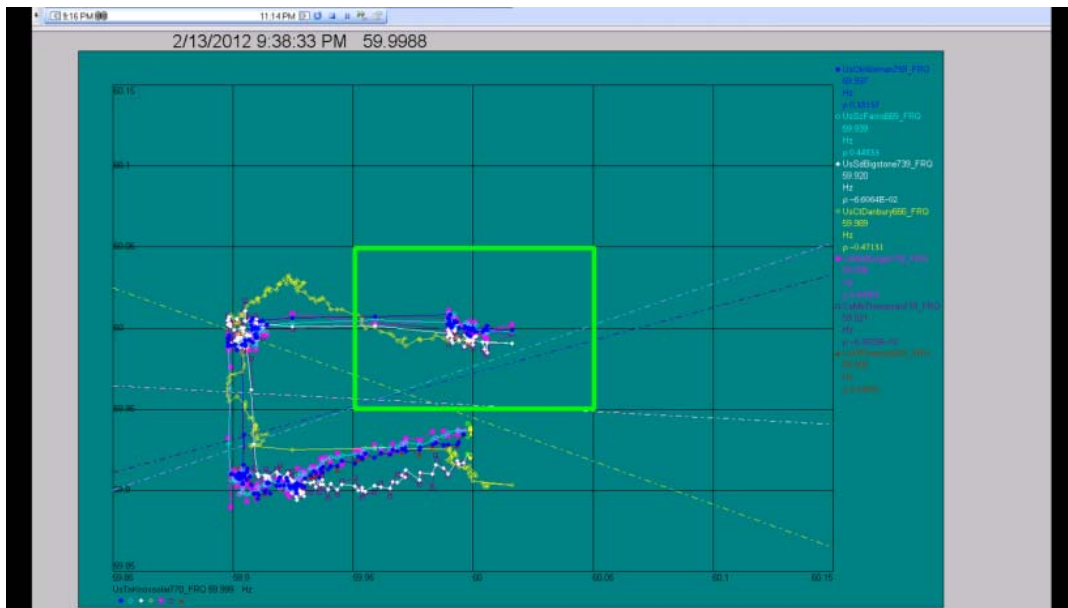


Fig. 5: Eastern Interconnection Oscillations

Another observation of continuum behavior of the grid is shown in Fig. 5. This is from the NASPI Visualization meeting in Orlando Florida, in February 2012. Videos are available on Youtube via the NASPI website³. This is a plot of synchronous frequencies throughout the Eastern Interconnection plotted against the master station located in Knoxville, TN.

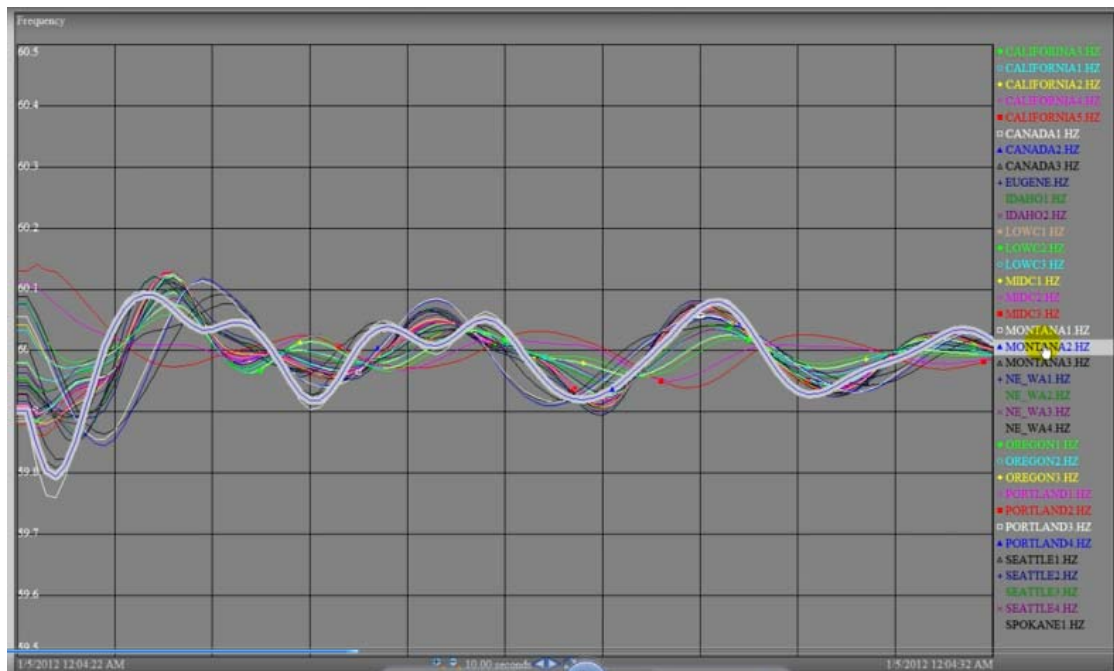


Fig. 6: Wave propagation throughout WECC

³<http://www.youtube.com/playlist?list=PLBA78489073233D0C>

This shows that the EI appears to be operating in two loosely connected grids in a four minute limit cycle. These data are from the FNET system and were provided by University of Tennessee for the visualization demonstrations.

Another example from the NASPI Visualization conference is shown in Fig. 6. The trend lines show the behavior of the WECC as a result of a series capacitor fault occurring in Montana. The out of phase peaks show wave propagating through the WECC based on this fault.

A fourth observation showing that the wide area grid can be modeled as a continuum system can be deduced from data collected during the Chief Joseph Brake tests. The WECC operates the Chief Joseph Brake located in the north central part of the state of Washington as part of studies to determine grid stability. These events can be measured by PMUs located in the WECC.

PMUs located in San Leandro, CA and in La Jolla, CA were used to measure the time of arrival of the waveform from two of the Chief Joseph events in October of 2012. We observed travel times of 261 mS and 305 mS between San Leandro and La Jolla on these events. For other events over the summer of 2012, such as generator trips, we observed transient times averaging about 500 mS between these two measurement locations.

The waveforms observed at San Leandro and at La Jolla are shown below in Figs 7 and 8. The waveforms follow the shapes commonly observed after forced outages of generation plants on the West coast, but with a smaller nadir. Most large generation trips cause a depression of frequency of more than 100 mHz. The Brake test consists of 1400 MW of power absorption for 500 mS, typically causing less than 50 mHz drop in observed frequency.

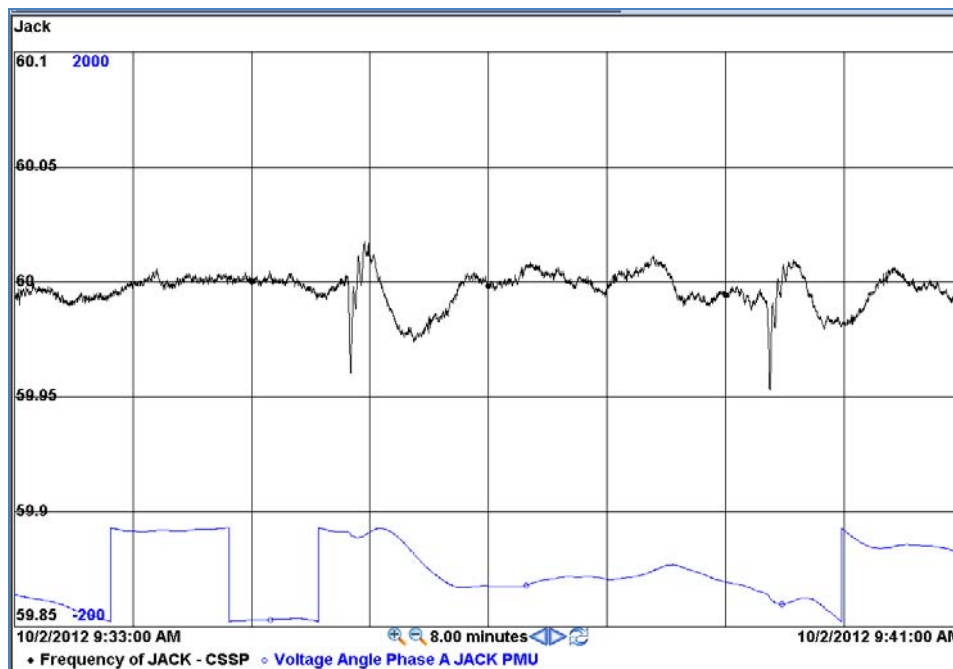


Fig. 7: Chief Joseph Brake test observations, October 2, 2012 observed at UCSD.

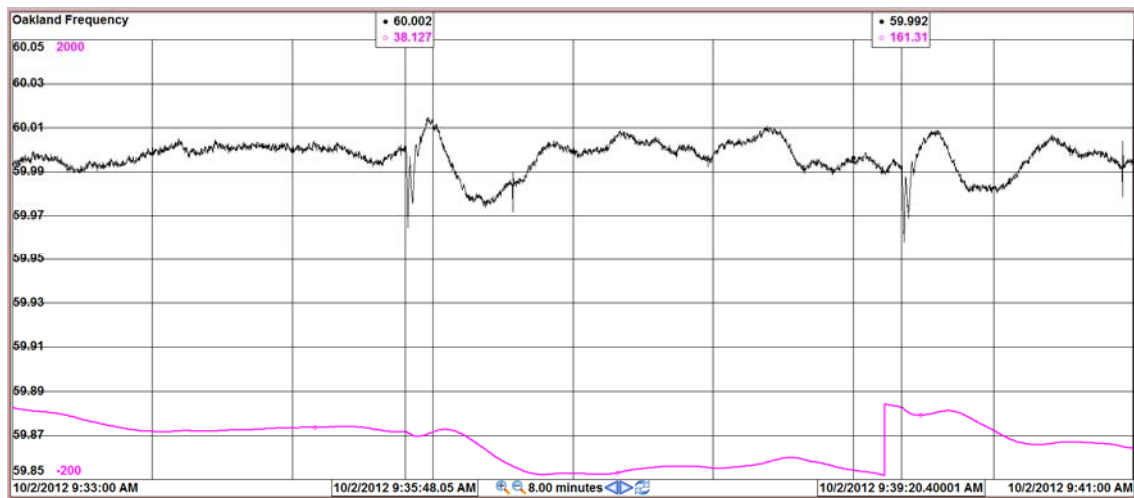


Fig. 8: Chief Joseph Brake tests Oct 2, 2012 observed at San Leandro, CA

These data show the existence of a wave and provide experimental evidence that the grid could be modeled as a continuum system.

For example the frequency disturbance caused by Chief Joseph Brake insertion is felt first in Washington State, then about one third of a second later in San Leandro, then 300 mS later in San Diego. The nadir of the disturbance is about 0.04 Hz. San Diego is about 14 degrees (0.04 Hz*360 degree/Hz) behind Chief Joseph one second after the brake insertion and depending on the location of the wrap, the subtraction could yield 374 degrees. Thus raw angles between these two stations should not be subtracted directly since they could be in different cycles.

We suggest angle distances should be calculated for each angle independently then subtracted to obtain the correct angle difference. This will insure they have same reference. If the angles are simply subtracted, or subtracted and then unwrapped, it is our conjecture, as shown below, that this yields incorrect information when operating in different cycles of wrapping.

In the case of a total blackout near Chief Joseph, the frequency would drop to zero; but, would remain near 60 Hz at San Diego. Since the electromechanical wave will take about one half second to travel a distance of 1300 miles, the instantaneous angle difference could be approximately (0-60) Hz *360 degrees/Hz*0.5 seconds or -10800 degrees or thirty cycles.

Larger disturbances caused by generator outages or sudden loss of load such as occurred during the San Diego Blackout of September 8, 2011 can create additional oscillation modes. During this event we observed oscillations at San Leandro at 5 Hz as well as the common modes in WECC of 0.3 and 0.7 Hz.

So the local protection systems operate within a local region where the system is synchronous and thus the common protection calculations apply. But for wide area protection this approach may not apply since the systems could be operating on different sections of the wave. These regions have been called “typals”⁴ in the Ilic and Zaborszky text. For wide area protection, it appears that using angle distance from the IEEE C37.118 reference is a more secure calculation. Chapter 4 of Liling’s thesis covers the topic of wide area protection in continuum systems.

This type of grid behavior has been observed in the Eastern Interconnection and was reported at the NASPI Visualization conference in Feb 2012, and is available on YouTube [5]

The IEEE C37.118.1-20116 synchrophasor standard defines angles as measured relative to an AC waveform represented by a 60 hertz (Hz) cosine wave with the peak occurring at the top of

⁴ Ilic, M and Zaborszky, J., Dynamics and Control of Large Electric Power Systems, Wiley Interscience, New York, 2000, p 396.

⁵ <http://www.youtube.com/playlist?list=PLBA78489073233D0C>

⁶ IEEE Std C37.118.1-2011, December 28, 2011.

second. Time accuracy must be better than $\pm 26 \mu$ seconds to achieve the accuracy specified in the standard (1 percent total vector error, TVE) with perfect measurement of voltage and current magnitudes (angles are computed from the digitized voltage and angle waveforms). So if the PMU has analog measurement errors, the time accuracy requirement decreases. This is one of the reasons the standard calls for 1 μ second time error in order to maintain the 1% TVE.

It is well known that grid frequency varies considerably from the nominal value over time. Consider a typical bus frequency of 60.05 Hz. Estimate how fast the angle of this bus will differ from the “reference” bus. The frequency difference from 60 Hz is 0.05 cycles per second faster so the angle will increase at the rate of:

$$\frac{0.05 \text{ cycles}}{\text{second}} \times \frac{360 \text{ degrees}}{\text{cycle}} = \frac{18 \text{ degrees}}{\text{second}}$$

and, if the frequency remains 0.05 Hz faster than nominal in 20 seconds the bus angle will be 360 degrees ahead of the reference.

Thus, at 21 seconds, the actual angle would be 378 degrees ahead of the reference waveform; but the PMU would report an angle of 18 degrees; however, the sine of these angles the same value (0.309). The sine and cosine functions of angle differences are used to compute real and reactive power flows.

Grid angles are actually smooth as measured from a common initial condition. The angle difference from the reference is lost by wrapping; but, a method of recovering this information is available by starting at common time to begin calculations. A careful look at the definition of phasors in the IEEE C37.118 illustrates this important point (Equation 5).

$$X(t) = \left(\frac{X_m(t)}{\sqrt{2}} \right) e^{j(2\pi f t + \phi)} \quad \text{Eq. (1)}$$

Where $X_m(t)$ is the time varying voltage or current magnitude, ϕ is the phase angle, and g is the offset in frequency from nominal. The initial condition of the integral is not defined explicitly but the end time is assumed to be (t) . We suggest midnight each day as the initial condition of the integral in the above equation

A typical voltage angle trend from a phasor measurement unit (PMU) meeting C37.118 standards is shown in Fig. 9. Data are measured from a 120 volt single phase power outlet. The vertical axis is the angle measurement in degrees from -200 degrees to +200 degrees. The horizontal axis is time. Two violet trend lines are shown: the thick dashed line is from channel A and the thin solid line is from channel B.

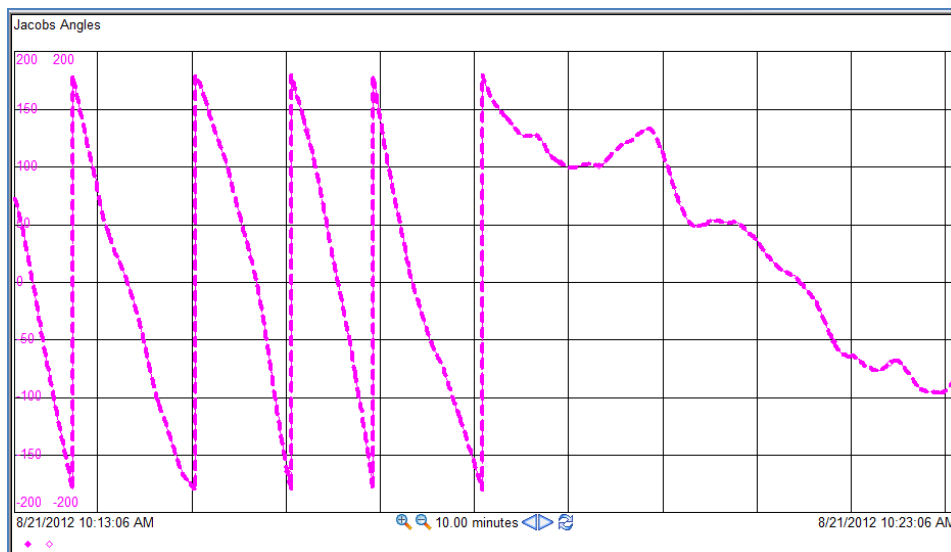


Fig. 9: Wrapped angle measurements over 10 minute intervals from 120 wall outlet.

In a ten minute period, the angle wrapped five times. The slope of the angle curve was negative over most of this interval meaning that frequency^[7] was below 60 Hz for most of the ten minute interval. During this interval, the angle decreased by more than $5 \times 360 = 1830$ degrees. In the latter part of the interval on the right hand side of Fig. 6, the rate of change of angle slows, and reverses itself becoming positive. This shows that frequency was above 60 Hz during this interval. Frequency is defined as the derivative of angle with respect to time thus:

$$f(t) = \frac{d\phi}{dt} \quad \text{Eq. (2)}$$

The derivative of angle must be a smooth function, but from Fig. 6, it is clear that the derivative is discontinuous. The reference model PMU in IEEE C37.118.1 computes its estimate of difference in frequency from nominal as a third order difference equation which is susceptible to noise and adds lag, so this method of estimating frequency will inject noise in the frequency estimate each time the angle wraps

Integrating both sides gives the formula for angle

$$\int_{t_0}^t f(t) dt = \int_{t_0}^t \frac{d\phi}{dt} dt = \phi(t) \quad \text{Eq. (3)}$$

The initial condition must be defined in order to determine the angle. In this case, t_0 could be midnight and the initial value of $\phi_0 = \phi(t_0)$. However rather than integrating frequency, we chose to

integrate $\frac{d\phi}{dt}$ using the first order difference to approximate the derivative over the past one sample interval. This provides a faster response than integrating frequency (see details of IEEE C37.118.1 sample frequency calculations for proof).

To further clarify, frequency and angle are plotted in Fig. 10. The scale of the frequency trend is from 59.9 Hz to 60.1 Hz. The scale of the angle trend is from -200 to +200 degrees. The green line shows the nominal frequency. When frequency is above 60 Hz, the slope of the angle trend is positive; and conversely, when frequency is below 60 Hz the slope is negative. Since frequency is a smooth function of time (unless there are faults), the angle must also be smooth. The discontinuous angles reported from PMUs range from $\pm \pi$ radians but in reality the angles continue to extend beyond this limit. One way estimate the accumulated angle is to integrate the rate of change of angle from an initial condition. The raw PMU angle is accumulated resulting in a smooth rate of change of angle.

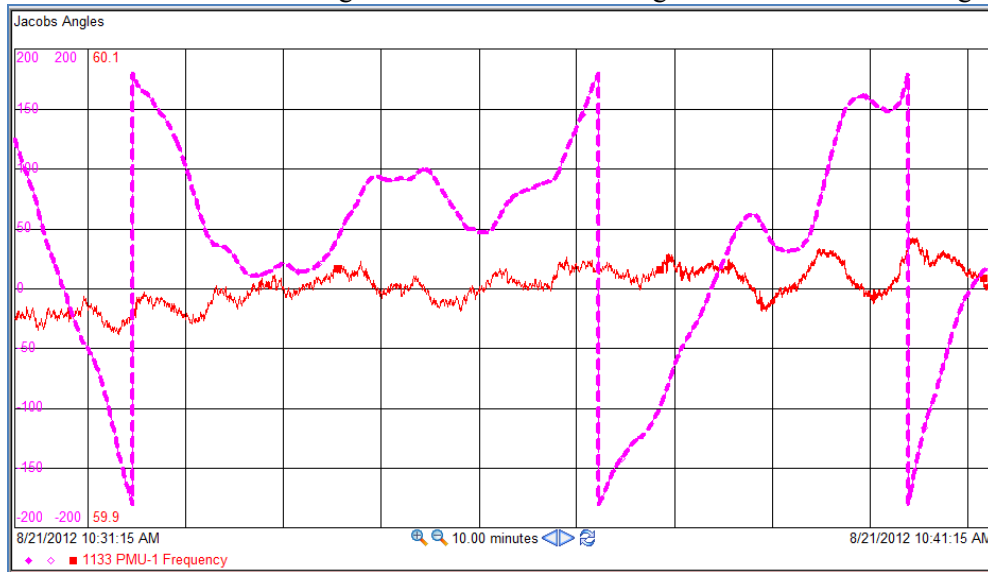


Fig. 10: Wrapped angle and frequency over ten minute intervals.

⁷ Frequency is defined as the rate of change of angle.

An example of unwrapped angle over a twenty-four hour interval is shown in Fig. 11.

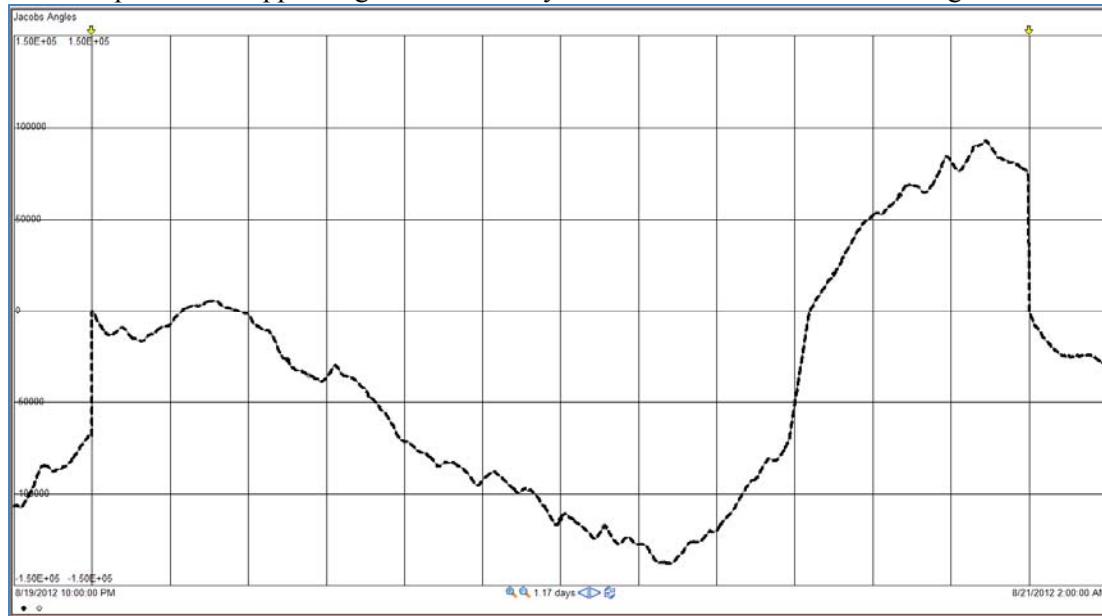


Fig. 11: Unwrapped angle over a 24 hour period.

The vertical axis represents unwrapped angle from -150,000 to +150,000 degrees. Perhaps a better way to display the magnitude is in decimal cycles, for example 100,000 degrees would be 277.77 cycles or could also be reported as time error as 4.629 seconds. At exactly midnight each day, the unwrapped angle is re-initialized to the current value of its wrapped angle. The unwrapped angle varies from large negative values to large positive values over a one day period. The yellow down arrows show that substitutions⁸ were made (the re-initialization).

The vertical axis is multi-scaled to show unwrapped angle and frequency. The green horizontal line represents nominal frequency (60Hz). Note that if the grid frequency is exactly 60 Hz, both the wrapped angle and the unwrapped angle are constant. Just before midnight on August 20, 2012, the unwrapped angle was 75,178 degrees. Since, the IEEE C37.118 reference angle is zero; the grid was operating 3.48046 seconds faster than the reference as defined by the C37.118 standard.

This is computed as follows:

$$\frac{75,178 \text{ degrees}}{360 \text{ degrees/cycle}} \times \frac{60 \text{ cycles}}{\text{second}} = 3.480463 \text{ seconds (fast)}$$

At the first sample after midnight, the value of the unwrapped angle is set to the current value of the wrapped angle. This is done for all angles being measured and sent to the PI System. The reset function can be applied at any time, not just at midnight. For example the WECC may use another time base as the reference, for example 7:00 AM. The main reason for re-initialization of the unwrapped angle calculation is to accommodate adding new PMUs or PMUs restarting during the day. A new PMU coming on-line is marked as “bad” data until it is initialized at the same time as other PMUs in the Balancing Authority [9].

2.1 Angle Distance

An example of the angle distance calculated over a period of one week is shown below in Fig. 12.

⁸ This is a built-in PI ProcessBook function

⁹ NERC Balancing Authority

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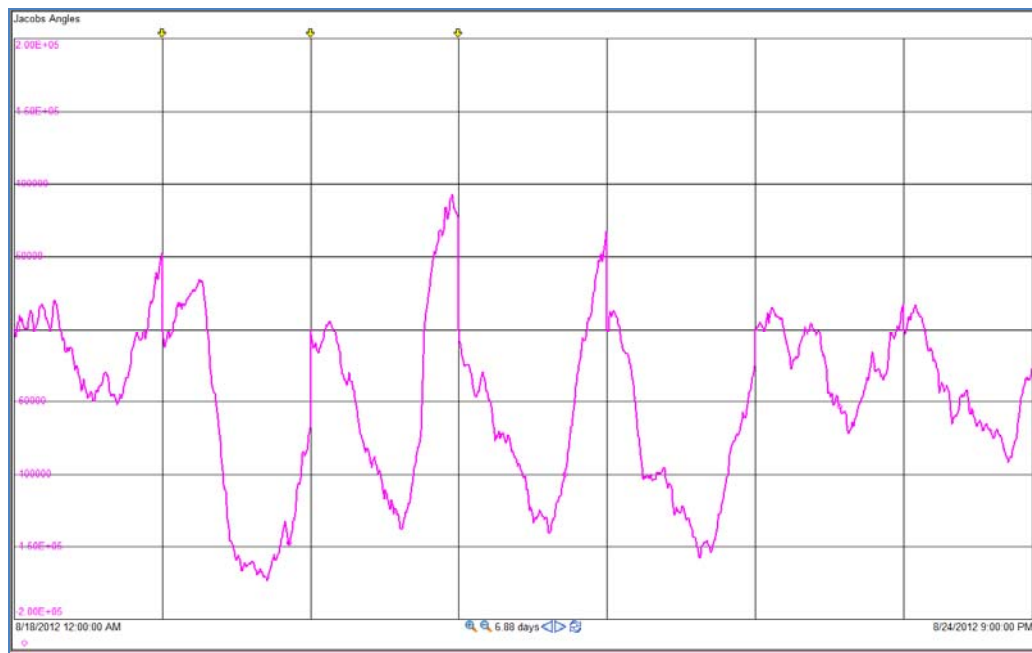


Fig. 12: Angle distance over a one week period

This shows the unwrapped angle trends for Saturday thru Friday at 9 pm. Note that the angle was below -150,000 degrees for a part of the Sunday and Wednesday. Our conjecture is that lower unwrapped angles imply greater grid stress. Note this also means that the average frequency was lower than 60 Hz which implies higher load and deficiency in energy in the grid. In this example, Sunday through Wednesday was hotter than Thursday and Friday.

2.2 Unwrapping Order

This section illustrates that angular distance values differ

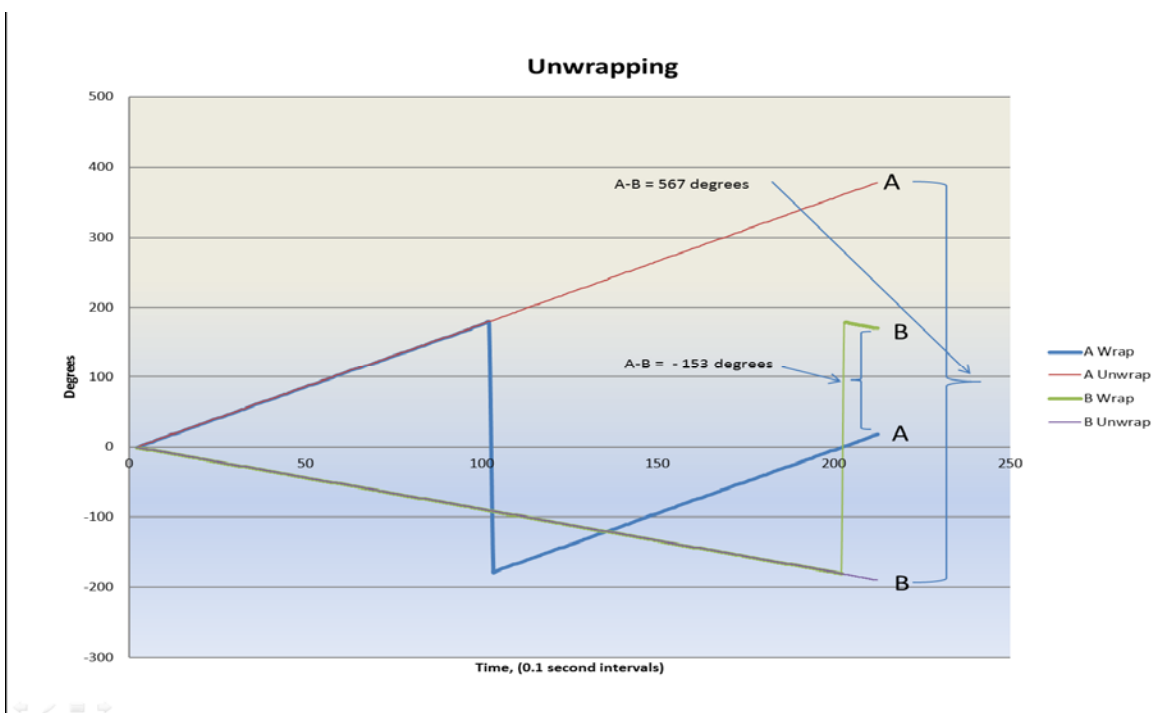


Fig. 13: comparisons of wrapping and unwrapped angle differences

whether one unwraps then subtracts or subtracts then unwraps.

The vertical axis is in degrees from ± 300 degrees and the horizontal axis is time in units of 0.1 seconds, so 200 means 20 seconds. Consider two buses, A and B that are located at least several hundred miles apart and are not directly connected by a single tie line. The buses are connected via a mesh network.

Bus A starts at zero angle, with jump in frequency to 60.05 Hz at time zero, so will have a rate of change of 18 degrees per second. Bus B starts at zero angle, with jump in frequency to 59.975 Hz at time zero, so will have a rate of change of -9 degrees per second.

Twenty-one seconds later: (this is not an unreasonable assumption by referring to Fig. 2, where frequency exceeds 60.05 for several minutes). Bus A = 18 degrees (one wrap); Bus B = 171 degrees (one wrap); Difference between Bus A and Bus B

$$A - B = 18 - (171) = -153 \text{ degrees}$$

No further unwrapping is required since the difference is inside $\pm \pi$. If the difference were outside this range, ± 360 degrees would be added to the difference.

In the case of unwrapping first, then subtracting, then twenty-one seconds later: Bus A = 378 degrees, Bus B = -189 degrees. Difference between bus A and Bus B:

$$A - B = 378 - (-189) = 567 \text{ degrees}$$

Note that both angle differences are in the third quadrant and have the same trigonometric values:

$$\begin{aligned} \cos(567 \text{ degrees}) &= -0.89101 \\ \cos(-153 \text{ degrees}) &= -0.89101 \end{aligned}$$

$$\begin{aligned} \sin(567 \text{ degrees}) &= -0.45399 \\ \sin(-153 \text{ degrees}) &= -0.45399 \end{aligned}$$

Thus the driving force indicated by the angle difference of -153 degrees or the angle difference of 567 degrees is the same. However, the convention for direction of power flow is from high angle to low angle. Thus the two methods give different indications of flow direction. By convention, if A-B is positive, the flow is toward B. Thus the former method gives the correct conventional flow direction, whereas the latter method provides the incorrect sign. This shows numerically that subtraction of angles is not commutative. That is the order of subtraction is important.

Two methods of computing angle differences are described. Both give the same trigonometric result, but the resulting angular distances are different. The method of unwrapping angles first then subtracting to get the angle difference appears to provide more information about the grid than subtracting angles first then unwrapping. The first method also provides the correct sign for the conventional power flow direction.

The first method also provides the means to determine the system time error but periodic re-initialization is recommended. The second method requires no initialization, but does not provide time error or indication of grid stress or the correct flow direction.

The first method may provide an indication of grid stress by measuring the electrical distance from the reference standard waveform. The angle distance can also be displayed on 3D surface charts where the x axis represents latitude, y axis represents longitude, and the z axis represents angle distance. This type plot will show operators the multiple directions and magnitude of power flowing in the service area.

3 CONCLUSION

We show that the grid behaves as a continuous system based on synchronized measurements at different locations in the grid. The grid can be described more accurately using partial differential equations rather than differential algebraic equations. We also show that computation of angle differences should be done by first unwrapping the angle with respect to an initial condition and then performing the subtraction. The order of the operations are important since they are not commutative.

REFERENCES

- [1] IEEE Std C37.118.1-2011 (Revision of IEEE Std C37.118-2005, IEEE 3 Park Avenue, New York, New York, 10016-5997, 28 December 2011..